THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

5a. Functional Dependencies

CSCI 2541 Database Systems & Team Projects

Wood

Slides adapted from Profs. Bhagi Narahari and Rahul Simha, and book by Silberschatz, Korth, and Sudarshan

Good and Bad Schemas

Functional Dependencies

Normal Forms based on Functional Dependencies

Normal Form Examples: 1NF

1NF: Attributes should be atomic and tables should have no repeating groups

Customer ID	First Name	Surname	Telephone Number
123	Pooja	Singh	555-861-2025, 192-122-1111
456	San	Zhang	(555) 403-1659 Ext. 53; 182-929-2929
789	John	Doe	555-808-9633

BAD: One cell should not have multiple values in it!

Customer ID	First Name	Surname	TNumber1	TNumber2
123	Pooja	Singh	555-861-2025	192-122-1111
456	San	Zhang	(555) 403-1659 Ext. 53	182-929-2929
789	John	Doe	555-808-9633	

BAD: Should not have repeating columns!

Normal Form Examples: 2NF

2NF: No value in a table should be dependent on only **part** of a key that uniquely identifies a row

1NF

Customer ID	First Name	Surname	Telephone Number
123	Pooja	Singh	555-861-2025
123	Pooja	~~Singh~~	192-122-1111
456	San	Zhang	182-929-2929
456	San	Zhang	(555) 403-1659 Ext. 53
789	John	Zhang	555-808-9633

BAD: First/Last name depend on only Customer ID

VS

2NF

Customer ID	First Name	Surname
123	Pooja	Singh
456	San	Zhang
789	John	Zhang

Customer ID	Telephone Number
123	555-861-2025
123	192-122-1111
456	(555) 403-1659 Ext. 53
456	182-929-2929
789	555-808-9633

Meets 2NF

Normal Form Examples: 3NF

3NF: No value should be able to be dependent on another non-key field

<u>Customer ID</u>	First Name	Surname	Birthday	Age	Fav Color
123	Pooja	Singh	1/4/1984	37	Blue
456	San	Zhang	3/15/2001	19	Blue
789	John	Zhang	11/12/2006	14	Buff

BAD: Age is based on Birthday (non-key)

<u>Tournament</u>	<u>Year</u>	Winner	Winner's Birthplace
Indiana Invitational	1998	Al Fredrickson	Ohio
Cleveland Open	1999	Bob Albertson	New York
Des Moines Masters	1999	Al Fredrickson	Ohio
Indiana Invitational	1999	Chip Masterson	Kentucky

BAD: Birthplace based on Winner (non-key)

Summary

1NF: ensures atomicity of cells and prevents repetition of identical column types

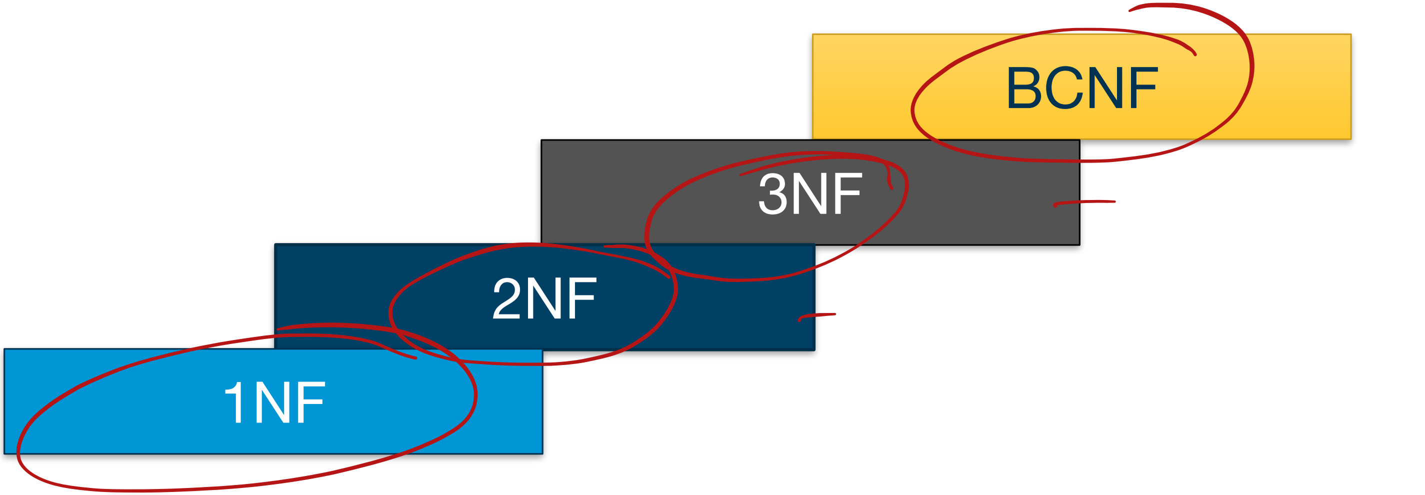
repetition of

2NF: prevents data across rows

3NF: prevents repetition of data within a row

Dependencies

How can we **formally represent dependencies** between Attributes in a Relation?



Functional Dependencies

Use **functional dependencies!** (abbreviated **FD**)

We say a set of attributes **X** functionally determines an attribute **Y** if *given the values of* ***X*** *we always know the only possible value of* ***Y***.

- Notation: **X** → **Y**
- X **functionally determines** Y
- Y is **functionally dependent** on X

X → Y

~~Name → GWID~~

Example:

- GWID → Name
- {GWID, CourseID, Semester, Year} → Grade

Sets of Functional Dependencies

Some more functional dependencies

- $\{GWID\} \rightarrow \{NAME, ADDRESS, MAJOR\}$
- $\{MAJOR\} \rightarrow \{DEPT\_NAME, DEPT\_CHAIR\}$

BS in CS

From above dependencies, we can infer

- $\{GWID\} \rightarrow \{DEPT\_NAME, DEPT\_CHAIR\}$

We can do math on functional dependencies!

> A functional dependency "holds" if it must be true for all legal relations

Functional Dependency Ops

Armstrong's Axioms: where A, B, C are sets of attributes

- **Reflexive rule**: if B ⊆ A, then A → B (if B is subset of A) {GWID, Major} → Major
- **Augmentation rule**: if A → B, then C A → C B
- **Transitivity rule**: if A → B, and B → C, then A → C

These rules are

- Sound and complete — generate all functional dependencies that hold.

F =

{GWID} → {Name, Address, Major}

{Major} → {Dept_Name, Dept_Chair}

{GWID, CourseID, Semester, Year} → Grade

Functional Dependency Ops

Armstrong's Axioms: where A, B, C are sets of attributes

- **Reflexive rule**: if B ⊆ A, then A → B (if B is subset of A)
- **Augmentation rule**: if A → B, then C A → C B
- **Transitivity rule**: if A → B, and B → C, then A → C

These rules are

- Sound and complete — generate all functional dependencies that hold.

Bonus rules to make life easier:

- **Union rule**: If α → β holds and α → γ holds, then α → β γ holds.
- **Decomposition rule**: If α → β γ holds, then α → β holds and α → γ holds.
- **Pseudotransitivity rule**: If α → β holds and γ β → δ holds, then α γ → δ holds.

Definition: Closure of a Set of FD's

Defn. Let F be a set of FD's.

Its **closure, F+,** is the set of all FD's:

$$\{X \rightarrow Y \mid X \rightarrow Y \text{ is derivable from F by Armstrong's Axioms}\}$$

Two sets of dependencies F and G are equivalent if F+=G+

- i.e., their closures are equal
- i.e., the same sets of FDs can be inferred from each

Example Closure

What FDs can we infer?

R = (A, B, C, G, H, I)

F = { A → B
A → C
CG → H
CG → I
B → H}

Reflexive rule: if β ⊆ α, then α → β

Augmentation rule: if α → β, then γ α → γ β

Transitivity rule: if α → β, and β → γ, then α → γ

A → H

A → BC

CG → HI

AG → HI

CBG → HI

A → A

AG → BI

Example Closure

$R = (A, B, C, G, H, I)$

$$F = \{ A \rightarrow B \\ A \rightarrow C \\ CG \rightarrow H \\ CG \rightarrow I \\ B \rightarrow H \}$$

Reflexive rule: if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$

Augmentation rule: if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$

Transitivity rule: if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$

A few members of F+ include:

- $A \rightarrow H$
  - by transitivity from $A \rightarrow B$ and $B \rightarrow H$
- $AG \rightarrow I$
  - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
- $CG \rightarrow HI$
  - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity

Functional Dependencies and Keys

A Candidate Key is a minimal set of attributes which are sufficient to uniquely identify each tuple in a relation

- All other attributes must be functionally dependent on the set of attributes that make up the Candidate Key.

Thus a candidate key must be a minimal set of attributes which can appear on the left hand side of functional dependencies, but will produce a closure that includes all other attributes on the right hand side

A candidate key must be a minimal set of attributes which can appear on the left hand side of functional dependencies, but will produce a closure that includes all other attributes on the right hand side

What is a candidate key for R?

$R = (A, \cancel{B}, \cancel{C}, G, \cancel{H}, \cancel{I})$

$$
\begin{aligned}
F = \{\; & A \rightarrow \cancel{B} \\
& A \rightarrow \cancel{C} \\
& CG \rightarrow H \\
& CG \rightarrow I \\
& \cancel{B} \rightarrow \cancel{H} \;\}
\end{aligned}
$$

A, G

A candidate key must be a minimal set of attributes which can appear on the left hand side of functional dependencies, but will produce a closure that includes all other attributes on the right hand side

What is a candidate key for R?

R = (A, B, C, G, H, I)

$$
\begin{aligned}
F = \{\ A &\rightarrow B \\
A &\rightarrow C \\
CG &\rightarrow H \\
CG &\rightarrow I \\
B &\rightarrow H\}
\end{aligned}
$$

Candidate Key: (A, G)
Non-Prime Attribs:
(B,C,~~G~~,H,I)

Good and Bad Schemas

Functional Dependencies

Normal Forms based on Functional Dependencies

Redefining 2NF

Using Functional Dependencies and Closures lets us more precisely define our Normal Forms

Second Normal Form: For every **X** → **A** that holds over relationship schema **R**, where A is a non-prime attribute (i.e., A is not an attribute in any candidate key)

$AB \rightarrow A$

1. either **A** ∊ **X** (it is trivial), or
2. **X** is a superkey for **R**, or
3. **X** is transitively dependent on a super key R

Easier to think of the opposite: There cannot be **X** → **A** where X is a partial candidate key for R

- Says nothing about non-prime to non-prime dependencies!

2NF Violations

I F S T

(1)

ID	First Name	Surname	Telephone Number
123	Pooja	Singh	555-861-2025
123	Pooja	Singh	192-122-1111
456	San	Zhang	182-929-2929
456	San	Zhang	(555) 403-1659 Ext. 53
789	John	Zhang	555-808-9633

843 John Zhang

✗ 2NF

$I \rightarrow F, S$

$IT \rightarrow F, S$

(2)

Tournament	Year	Winner	~~Winner's~~ Birthplace
Indiana Invitational	1998	Al Fredrickson	Ohio
Cleveland Open	1999	Bob Albertson	New York
Des Moines Masters	1999	Al Fredrickson	Ohio
Indiana Invitational	1999	Chip Masterson	Kentucky

$TY \rightarrow W$

$W \rightarrow B$

$YT \rightarrow B$

✓ 2NF

2NF Violations

ID	First Name	Surname	Telephone Number
123	Pooja	Singh	555-861-2025
123	Pooja	Singh	192-122-1111
456	San	Zhang	182-929-2929
456	San	Zhang	(555) 403-1659 Ext. 53
789	John	Zhang	555-808-9633

ID -> {**First Name, LastName**} Violates 2NF since **ID** is a partial Candidate Key

Tournament	Year	Winner	Winner's Birthplace
Indiana Invitational	1998	Al Fredrickson	Ohio
Cleveland Open	1999	Bob Albertson	New York
Des Moines Masters	1999	Al Fredrickson	Ohio
Indiana Invitational	1999	Chip Masterson	Kentucky

No 2NF violation

Redefining 3NF

Third Normal Form (3NF): For every **X** → **A** that holds over relationship schema **R**,

1. either **A** ∈ **X** (it is trivial), or
2. **X** is a superkey for **R**, or
3. **A** is a member of some candidate key for **R**

Easier to think of: **X** must be a full candidate key, unless A itself is a part of a candidate key

"Every non-key attribute must provide a fact about the Key, the whole Key, and nothing but the Key… so help me Codd"

3NF Violations

C → F, S, B, A, F

B → A X3NF

<u>Customer ID</u>	First Name	Surname	Birthday	Age	Fav Color
123	Pooja	Singh	1/4/1984	37	Blue
456	San	Zhang	3/15/2001	19	Blue
789	John	Zhang	11/12/2006	14	Buff

T Y → W B

W → B

~~3NF~~

<u>Tournament</u>	<u>Year</u>	Winner	~~Winner's~~ Birthplace
Indiana Invitational	1998	Al Fredrickson	Ohio
Cleveland Open	1999	Bob Albertson	New York
Des Moines Masters	1999	Al Fredrickson	Ohio
Indiana Invitational	1999	Chip Masterson	Kentucky

3NF Violations

C -> F,S,B,A,F
B->A

<u>Customer ID</u>	First Name	Surname	Birthday	Age	Fav Color
123	Pooja	Singh	1/4/1984	37	Blue
456	San	Zhang	3/15/2001	19	Blue
789	John	Zhang	11/12/2006	14	Buff

Birthday->Age holds, but **Birthday** is not a superkey

T,Y -> W, WB
W -> WB

<u>Tournament</u>	<u>Year</u>	Winner	Winner's Birthplace
Indiana Invitational	1998	Al Fredrickson	Ohio
Cleveland Open	1999	Bob Albertson	New York
Des Moines Masters	1999	Al Fredrickson	Ohio
Indiana Invitational	1999	Chip Masterson	Kentucky

Winner -> Birthplace holds, but **Winner** is not a superkey

Normal Forms 1-3

1NF: Attributes should be atomic and tables should have no repeating groups

- *Prevents messiness within a cell and repetition of rows*

2NF: There cannot be **X → A** where X is a partial candidate key for R

- Doesn't forbid non-prime to non-prime dependencies
- *Prevents repetition of cells across rows*

3NF: There cannot be **X → A** where X is not a full candidate key for R (unless A is a Key)

- Only allows dependencies on Keys
- *Prevents repetition of data within a row*

Good and Bad Schemas

Functional Dependencies

Even more normal forms!

Normal Forms 1-3

1NF: Attributes should be atomic and tables should have no repeating groups

- *Prevents messiness within a cell and repetition of rows*

2NF: There cannot be **X → A** where X is a partial candidate key for R

- Doesn't forbid non-prime to non-prime dependencies
- *Prevents repetition of cells across rows*

3NF: There cannot be **X → A** where X is not a full candidate key for R (unless A is a Key)

- Only allows dependencies on Keys
- *Prevents repetition of data within a row*

Normal Form

Normal form reference:

- 2NF: Cannot have partial Key on left hand side (LHS)
- 3NF: Meet 2NF and LHS must be full Candidate Key or RHS must be a key

ID	First name	Cid	Subj	Num	Grad
1	Sam	570103	SW	cs143	B
23	Dan	550103	DB	cs178	A

Functional Dependencies

ID → FirstName

ID, Cid → Num, Grade

Num → Subj

X 2NF

What normal form is this?

Normal Form

Normal form reference:

- 2NF: Cannot have partial Key on left hand side (LHS)
- 3NF: Meet 2NF and LHS must be full Candidate Key or RHS must be a key

<u>ID</u>	First name	<u>Cid</u>	Subj	Num	Grad
1	Sam	570103	SW	cs143	B
23	Dan	550103	DB	cs178	A

Functional Dependencies

ID → FirstName partial key ID violates 2NF!

ID, Cid → Num, Grade

Num → Subj non-prime LHS would also violate 3NF!

Only meets 1NF

How to Judge Decomposition?

R

R1 R2

X

ID	First name	Cid	Subj	Num	Grad
1	Sam	570103	SW	cs143	B
23	Dan	550103	DB	cs178	A

F
- ID → FirstName
- ID, Cid → Num, Grade
- Num → Subj

R_1 = ID, Name, Cid

R_2 = CID, Subj, Num, Grade

② CID → Subj, Num, Grade

① CID → ID, Name

Lossless Decomposition test:

- **R1**, **R2** is a lossless join decomposition of **R** with respect to **F** **iff** at least one of the following dependencies is in **F+**

① **(R1 ∩ R2) → R1 – R2**

② **(R1 ∩ R2) → R2 – R1**

$R_1 \cap R_2$ = CID

$R_1 - R_2$ = ID, Name

$R_2 - R_1$ = Subj, Num, Grade

Lossless Decomposition

R1 R2

ID	First name	Cid	Subj	Num	Grad
1	Sam	570103	SW	cs143	B
23	Dan	550103	DB	cs178	A

F

ID → FirstName
ID, Cid → Num, Grade
Num → Subj

F_1 ID → Name

F_2 Num → Subj

R1 ∩ R2 = CID

R1 - R2 = ID, First

R2 - R1 = Subj, Num, Grade

Not lossless!

Lossless Decomposition test:

- **R1**, **R2** is a lossless join decomposition of **R** with respect to **F** **iff** at least one of the following dependencies is in **F+**
- **(R1 ∩ R2) → R1 – R2**
- **(R1 ∩ R2) → R2 – R1**

Dependency Preservation

We also must maintain dependences

After decomposition from **R** to **R1 … Rn**, the closure of FDs of all **R1…Rn** must be equivalent to that of **R**

R1 = ID, FirstName, CID

R2 = CID, Sub, Num, Grade

BAD

or

R3 = ID, FirstName

R4 = ID, CID, Sub, Num, Grade

Good

ID → FirstName

ID, Cid → Num, Grade

Num → Subj

Dependency Preservation

We also must maintain dependences

After decomposition from **R** to **R1 … Rn**, the closure of FDs of all **R1…Rn** must be equivalent to that of **R**

R1 = ID, FirstName, CID

R2 = CID, Sub, Num, Grade

or

R3 = ID, FirstName

R4 = ID, CID, Sub, Num, Grade

ID → FirstName

ID, Cid → Num, Grade

Num → Subj

R1,R2 will lose the ID,CID -> Num, Grade FD

R3,R4 will maintain all FDs

3NF

It is **always possible** to decompose a relation R into a set of relations R1…Rn which is **dependency preserving** and **lossless** that is in 3NF

3NF is the baseline for acceptable DB normalization in practice!

but 3NF is not perfect…

When does 3NF fail?

Suppose we want to store addresses:

ADDR_INFO(<u>**CITY**</u>, <u>**ADDRESS**</u>, <u>**ZIP**</u>)

{CITY, ADDRESS} → ZIP

{ZIP} → {CITY}

Meets 3NF since LHS is a full Key **or** RHS is a Key

> 3NF: There cannot be **X → A** where X is not a full candidate key for R **(unless A is a Key)**

When does 3NF fail?

ADDR_INFO(**CITY**, **ADDRESS**, **ZIP**)

{CITY, ADDRESS} → ZIP

{ZIP} → {CITY}

City	Address	Zip
Washington	800 22nd St NW	20052
Washington	1600 Pennsylvania Avenue NW	20050
Philadelphia	Main St	20050

3NF: There cannot be **X → A** where X is not a full candidate key for R **(unless A is a Key)**

When does 3NF fail?

ADDR_INFO(**<u>CITY</u>**, **<u>ADDRESS</u>**, **ZIP**)

{CITY, ADDRESS} → ZIP

{ZIP} → {CITY}

City Zip

City	Address	Zip
Washington	800 22nd St NW	20052
Washington	1600 Pennsylvania Avenue NW	20050
Philadelphia	101 South Street	20050

3NF does not prevent insertion/update of tuples which violate our FDs!

3NF vs BCNF

Third Normal Form (3NF): For every $\mathbf{X} \rightarrow \mathbf{A}$ that holds over relationship schema **R**,

1. either $\mathbf{A} \in \mathbf{X}$ (it is trivial), or
2. **X** is a superkey for **R**, or
3. **A** is a member of some key for **R**

Option 3 can result in update anomalies!

Boyce-Codd Normal Form (BCNF) resolves this issue:

For every $\mathbf{X} \rightarrow \mathbf{A}$ that holds over relationship schema **R**,

1. either $\mathbf{A} \in \mathbf{X}$ (it is trivial), or
2. **X** is a superkey for **R**

BCNF

BCNF is stricter than 3NF

- If a relation is in BCNF, it is also in 3NF;
- if it is not in 3NF, it is not in BCNF

Note:

- There are polynomial time algorithms **guaranteed to provide a lossless, dependency preserving decomposition** into 3NF
- **but** a **dependency preserving** decomposition into BCNF **may not exist**, and no polynomial time algorithm for **lossless decomposition** is known.

Normalization Summary

Functional Dependencies: Capture the dependencies between attributes

Normalization: Provides a schema that ensures functional dependencies will be kept consistent, without losing data

Normal Forms: Try to achieve BCNF, but 3NF is OK in some cases (1NF/2NF -> bad design!)

2NF vs 3NF vs BCNF

Second Normal Form: For every **X** → **A** that holds over relationship schema **R**,

1. If **A** is a non-prime attribute, then **X** cannot be a partial Candidate Key

Third Normal Form (3NF): For every **X** → **A** that holds over relationship schema **R**,

1. either **A** ∈ **X** (it is trivial), or
2. **X** is a superkey for **R**, or
3. **A** is a member of some key for **R**

Option 3 can result in update anomalies!

Boyce-Codd Normal Form (BCNF) resolves this issue:

For every **X** → **A** that holds over relationship schema **R**,

1. either **A** ∈ **X** (it is trivial), or
2. **X** is a superkey for **R**